This section contains problems intended to challenge students and teachers of college mathematics. We urge you to participate actively BOTH by submitting solutions and by proposing problems that are new and interesting. To promote variety, the editors welcome problem proposals that span the entire undergraduate curriculum.

**Proposed problems** should be sent to Curtis Cooper, either by email as a pdf, TeX, or Word attachment (preferred) or by mail to the address provided above. Whenever possible, a proposed problem should be accompanied by a solution, appropriate references, and any other material that would be helpful to the editors. Proposers should submit problems only if the proposed problem is not under consideration by another journal.

**Solutions to the problems in this issue** should be sent to Shing So, either by email as a pdf, TeX, or Word attachment (preferred) or by mail to the address provided above, no later than February 15, 2011.

### PROBLEMS

936. Proposed by Cezar Lupu (student), University of Bucharest, Bucharest, Romania and Tudorel Lupu (student), Decebal High School, Constanta, Romania.

Let $a$, $b$, $c$ be three positive real numbers. Prove that

\[
a^3 + b^3 + c^3 + 2abc + \frac{9a^2b^2c^2}{(a+b+c)(a^2 + b^2 + c^2)} \geq ab(a+b) + bc(b+c) + ca(c+a).
\]


Let $a$, $b$, $c$ be the sides, $m_a$, $m_b$, $m_c$ the medians, $h_a$, $h_b$, $h_c$ the heights, $l_a$, $l_b$, $l_c$ the lengths of the angle bisectors and $R$ the circumradius of a triangle. Prove that

\[
\frac{l_a^4(m_a^2 - h_a^2)}{h_a^2(l_a - h_a)\sqrt{l_a \cdot h_a}} + \frac{l_b^4(m_b^2 - h_b^2)}{h_b^2(l_b - h_b)\sqrt{l_b \cdot h_b}} + \frac{l_c^4(m_c^2 - h_c^2)}{h_c^2(l_c - h_c)\sqrt{l_c \cdot h_c}} \geq 24R^2.
\]

938. Proposed by Ovidiu Furdui, Cluj, Romania.

Let $f : [0, 1] \to \mathbb{R}$ be an integrable function which is continuous at 1 and let $k \geq 1$. Find the value of

\[
\lim_{n \to \infty} \int_0^1 (x + 2^k x^2 + 3^k x^3 + \cdots + n^k x^n) f(x) \, dx.
\]
Let $A$ be an $n \times n$ complex matrix. Let $\text{adj}(A)$ be the adjugate of $A$, that is, the transpose of the matrix of the co-factors of $A$. Let $\text{tr}(A)$ be the trace of $A$. Show that $\text{tr}(\text{adj}(A)^k) = 0$ for all $k$, $1 \leq k \leq n$, if and only if $\text{adj}(A)^2$ is a zero matrix.

Proposed by Greg Oman, Ohio University, Athens OH.

Find all pairs $(R, n)$ where $R$ is a nonzero ring (not necessarily commutative or with identity) and $n > 1$ is an integer such that the following holds:

$$x_1x_2 \cdots x_n \in \{x_1, x_2, \ldots, x_n\}$$

for all $x_1, x_2, \ldots, x_n \in R$.

**SOLUTIONS**

**An externally trilinable 7-gon**

Proposed by Michael Scott McClendon, University of Central Oklahoma, Edmond OK.

Given an $n$-gon $P$, a point $x$ in the same plane as $P$ is said to be $m$-trilinable if there exist 3 points $x_1, x_2,$ and $x_3$ on $P$ such that $d(x, x_1) = d(x, x_2) = d(x, x_3) = m$, where $d(a, b)$ is the Euclidean distance between points $a$ and $b$. An $n$-gon is said to be externally trilinable if every point in the plane exterior to $P$ is $m$-trilinable for some $m$. Is it possible for a 7-gon to be externally trilinable?

Solution by Darryl Nester, Bluffton University, Bluffton OH.

First note that the statement “$x$ is $m$-trilinable” is equivalent to “a circle with radius $m$ centered at $x$ intersects $P$ in at least three points.”

Let $P$ be the non-convex hexagon $ABCDEF$ in Figure 1. Extend line segments $BA$ and $CD$ to meet at $G$, and $FA$ and $ED$ to meet at $H$. Then the exterior of $P$ is equal to the union of the interior of $\triangle EFH$ and the exterior of $\triangle BCG$. For a point $Z$ in the interior of $\triangle EFH$, we can clearly choose an appropriate radius so that a circle of that radius centered at $Z$ will intersect $P$ in at least three points. For example, if $Z$ is above $AD$, pick a point $K$ in the interior of the quadrilateral $AFED$, then the circle centered at $Z$ with radius $ZK$ intersects $P$ in at least three points. If $Z$ is in the interior of the

Figure 1. Non-convex hexagon