
PROBLEMS

BERNARDO M. ÁBREGO, *Editor*

California State University, Northridge

Assistant Editors: SILVIA FERNÁNDEZ-MERCHANT, California State University, Northridge; JOSÉ A. GÓMEZ, Facultad de Ciencias, UNAM, México; ROGELIO VALDEZ, Facultad de Ciencias, UAEM, México; WILLIAM WATKINS, California State University, Northridge

PROPOSALS

To be considered for publication, solutions should be received by March 1, 2011.

1851. *Proposed by Éric Pité, Paris, France.*

Let a be an arbitrary integer. Consider the recursive sequence of integers defined by $u_0 = 4$, $u_1 = 0$, $u_2 = 2$, $u_3 = 3$, and $u_{n+4} = u_{n+2} + u_{n+1} + a \cdot u_n$ for every integer $n \geq 0$. Prove that p divides u_p for every prime p .

1852. *Proposed by Radu Gologan, Institute of Mathematics “Simion Stoilow” of the Romanian Academy, Bucharest, Romania; and Cezar Lupu (student), University of Bucharest, Bucharest, Romania.*

Let $f : [0, 1] \rightarrow \mathbb{R}$ be a differentiable function with a continuous derivative such that $f(0) = f(1) = -\frac{1}{6}$. Prove that

$$\int_0^1 (f'(x))^2 dx \geq 2 \int_0^1 f(x) dx + \frac{1}{4}.$$

1853. *Proposed by Michael W. Botsko, Saint Vincent College, Latrobe, PA.*

A function f is continuous nearly everywhere if it is continuous on its domain except for a countable set. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a real-valued function.

- Prove that if f is continuous nearly everywhere, then for every open set $G \subseteq \mathbb{R}$ there are an open set O and a countable set C such that $f^{-1}(G) = O \cup C$.
- Is the converse of part (a) true? Prove or disprove.

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We invite readers to submit problems believed to be new and appealing to students and teachers of advanced undergraduate mathematics. Proposals must, in general, be accompanied by solutions and by any bibliographical information that will assist the editors and referees. A problem submitted as a Quickie should have an unexpected, succinct solution. Submitted problems should not be under consideration for publication elsewhere.

Solutions should be written in a style appropriate for this MAGAZINE.

Solutions and new proposals should be mailed to Bernardo M. Ábrego, Problems Editor, Department of Mathematics, California State University, Northridge, 18111 Nordhoff St, Northridge, CA 91330-8313, or mailed electronically (ideally as a L^AT_EX or pdf file) to mathmagproblems@csun.edu. All communications, written or electronic, should include **on each page** the reader's name, full address, and an e-mail address and/or FAX number.

1854. Proposed by Marian Tetiva, National College “Gheorghe Roşca Codreanu”, Bârlad, Romania.

Let n and d be nonnegative integers. Find the number of all subsets of $\{1, 2, \dots, n\}$ which do not contain two numbers whose difference is d . (Subsets with at most one element satisfy the condition by vacuity.)

1855. Proposed by Michael Goldenberg and Mark Kaplan, The Ingenuity Project, Baltimore Polytechnic Institute, Baltimore, MD.

Prove that the Euler line of a triangle is perpendicular to one of the medians, if and only if, the Brocard line is perpendicular to the symmedian line from the same vertex as the median.

(The Brocard line passes through the circumcenter and the symmedian point of a triangle. The symmedian point is the point of concurrency of the symmedians and a symmedian through a vertex is the line symmetric to the median with respect to the angle bisector from the same vertex.)

Quickies

Answers to the Quickies are on page 310.

Q1003. Proposed by Rick Mabry, Louisiana State University in Shreveport, Shreveport, LA.

What are the zeros of the n th derivative of $f(x) = x^2 e^x$?

Q1004. Proposed by Daniel Edelman (student), Mason–Rice Elementary School, Newton Centre, MA; and Alan Edelman, MIT, Cambridge, MA.

If we concatenate (run together) a finite sequence of nonzero numbers in base 10, can this number equal the product? For example, given 6, 54, and 321, we are comparing 654321 with $6 \cdot 54 \cdot 321 = 104004$ which is less than 654321.

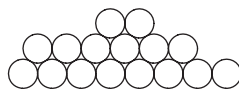
Solutions

Counting block fountains of coins

October 2009

1826. Proposed by Michael Woltermann, Washington & Jefferson College, Washington, PA.

A block fountain of coins is an arrangement of identical coins in rows such that the coins in the first row form a contiguous block, each row above that forms a contiguous block, and each coin in a higher row is supported by two adjacent coins in the row below. As an example,



If a_n denotes the number of block fountains with exactly n coins in the base, then $a_n = F_{2n-1}$, where F_k denotes the k th Fibonacci number. (Wilf, *generatingfunctionology*,