Fundamentals of Mathematics I: MA 160

Fall 2011

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Office hours: Tues. 2:00 – 3:30
Wed. 3:30 – 4:30
Thurs. 10:30– 11:45

Prerequisite

Satisfaction of the Westmont College mathematics proficiency requirement.

Course Objectives

The purpose of Math 160 is to promote a thorough understanding of the mathematics you should know to be qualified to teach at the elementary level. This includes:

> The base 10 number system and the standard algorithms for the basic operations;
>
> The algebraic properties of and the differences between the whole numbers, the integers, and the rational numbers and how the number systems are related;
>
> The crucial role played by logic and pattern recognition in mathematics;
>
> The importance of functions and pairing relationships – particularly one-to-one correspondences and equivalence relations;
>
> The use of variables and algebraic expressions to communicate and understand general procedures and relationships;

This course is not intended to correct deficiencies in basic mathematical background – you have demonstrated your basic competency in arithmetic and algebra by satisfying Westmont's mathematics competency requirement as a prerequisite to this course. Though this course will be sensitive to instructional issues, it is not intended to teach you how to teach – teaching methods will be addressed in courses from the Education Department. Rather, its purpose is to deepen your mathematical understanding. The goal of this course is to reconsider from a more advanced perspective the mathematical processes and ideas that one encounters during the early part of an academic career. Just as your reading abilities and appreciation of literature should far exceed the level of 'see spot run' or "Where the Red Fern Grows", so too one's knowledge and understanding of mathematics should extend beyond and be deeper than what one would be required to teach.

This course will examine familiar mathematics from a perspective that will be new to most of you. We will focus on the abstract and theoretical structure of mathematics, emphasizing problem solving, communication, and reasoning. Put another way, rather than practicing computation, we will be exploring and communicating the ideas and reasons behind the mathematical manipulations.

Relationship to other expectations

The *Mathematics Framework for California Public Schools (2005)* (http://www.cde.ca.gov/ci/ma/cf/) lists six goals for mathematics students.

1. that they develop fluency in computation;

2. that they learn to communicate mathematical ideas precisely;
3. that they develop logical thinking;
4. that they make connections among mathematical ideas and with other disciplines;
5. that they apply mathematics to everyday life;
6. that they develop an appreciation for the beauty and power of mathematics.

More recently, the National Governors Association Center for Best Practices and the Council of Chief State School Officers have been working on a Common Core State Standards Initiative. The final standards (http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf) were released in June 2010 and since have been adopted by 45 of the 50 states. California adopted the Common Core State Standards August 2, 2010. The standards identify the mathematical content that students should master at each grade level. For example, one of the outcomes for second graders is that they be able to "[e]xplain why addition and subtraction strategies work, using place value and the properties of operations." Third graders should be able to "[c]ompare two fractions with the same numerator or the same denominator by reasoning about their size." Moreover, at every level, students are expected to engage in a set of common mathematical practices.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

As elementary school teachers, you will be responsible for helping students and departments achieve these goals. Thus in addition to knowing the basic algorithms and manipulations of mathematics, you must become competent problem solvers, good communicators and be able to reason mathematically. You will need to see how mathematics builds on itself. If you are to teach your students to appreciate mathematics, you must learn to value mathematics yourself.

The Common Core State Standards use 10 major headings under which the outcomes for each grade level are organized. At most five of the major categories are addressed at any grade level. The major headings and the grade levels at which they are first introduced are listed below.

1. **Counting and Cardinality** (Grade K)
2. **Operations and Algebraic Thinking** (Grade K)
3. **Number and Operations in Base Ten** (Grade K)
4. Measurement and Data (Grade K)
5. Geometry (Grade K)
6. **Number and Operations – Fractions** (Grade 3)
7. **Ratios and Proportional Relationships** (Grade 6)
8. **The Number System** (Grade 6)
9. **Expressions and Equations** (Grade 6)
10. Statistics and Probability(Grade 6)

MA 160 addresses the seven major headings which are listed in bold. The remaining three are taken up in MA 165.

It is important to recognize that mathematics, and so this course, does not easily split up into simple categories. Just try doing measurement, probability, statistics, or algebra without understanding numbers.

The ideas underlying numbers depend in turn upon patterns, functions, logic, and are often best understood in terms of geometry and measurement. In general, mathematics consists of the search for patterns and the logical analysis of systems and structures. Geometry is meaningless until one begins to understand its patterns and structure via logic, but there are no patterns or structures to understand without some elementary geometric ideas in the first place.

It is interesting to note that the areas least likely to be named as part of mathematics, namely patterns and logic, play such a central role in mathematics. Unfortunately, many students come away from mathematics lessons and courses believing mathematics to be an activity where students are to drill on computational techniques and instructors demonstrate the techniques and check answers. While some drill is needed, the driving force in mathematics should always be towards conceptual understanding. The value of understanding is clearly evident in the Common Core's list of Mathematical Practices.

You will find that this course is structured to provide you with the experience of developing your own mathematical ideas. You will work on exercises that will lead you toward understanding of concepts behind familiar operations as well as some new ideas. Your instructor will be available to help you when you when you get stuck and to provide an over-arching context for what you are doing. The important point is that you will be doing rather than listening to lectures.

Course Goals and General Education:

MA 165 satisfies the GE Common Contexts requirement *Reasoning Abstractly*. A well educated student should be able to engage in critical and analytical reasoning about non-empirical, abstract concepts, issues, theories, objects and structures. The ability to look beyond the particulars of a situation or problem and recognize the common characteristics that fit a particular instance into an overarching structure is a valuable skill. The ability to work out the implications of clearly identified assumptions and definitions is another critical skill. This type of thinking enables one to understand and evaluate abstract arguments and explanations, analyze abstract concepts and solve abstract problems. This skill, in turn, releases one from the need to solve every individual problem from scratch when it is encountered.

MA 160 is filled with examples of this type of thinking. Almost every calculation one encounters is enabled by connecting a particular problem to a general scheme or principle. Moreover, mathematics as a discipline constantly challenges the user to answer the question: Why does this work? This question forces the diligent student to think abstractly about the principles involved and to supply a general (or abstract) argument that explains the situation.

A constantly recurring theme in this course will be the use of analytic proof to verify properties of various types of numbers. One of the primary characteristics of mathematics is its insistence on general verification of claims. In mathematics it never suffices to give examples. Examples may illustrate an idea, may suggest a relationship, or may hint at a line of reasoning, but they can never establish a general conclusion. As this course progresses, you will see how one type of mathematical structure is derived from another and how the properties of one type of number derive from another. Indeed, one could say that this progression and development is the core of the course. By the time we finish, you should have shifted your view of mathematics (and arithmetic in particular) from understanding it as a set of manipulative procedures to seeing it as a set of powerful, related structures whose properties can be proved formally.

Conceptual (Content) Goals:

Students in Mathematics 160 should be able to reason, solve problems, and communicate effectively in the following areas:

1. Logic;
2. Set Theory;
3. Functions and relations;

4. The algebraic structure of the whole numbers and the integers;
5. The addition, subtraction, multiplication, and division algorithms;
6. Various approaches to the representation of numbers and the effects on the algorithms.
7. Number theory;
8. The construction of and the differences between the whole numbers, the integers, and the rational numbers;
9. Recognition and communication of mathematical patterns.

Affective (Non-Content) Goals:

Students in Mathematics 160 should:

1. Demonstrate a deepened understanding of the nature of mathematics by naturally asking and attempting to answer questions about how and why mathematical operations work;
2. Value cooperative learning and see it modeled;
3. Acquire a positive attitude towards and a lack of fear of mathematics.

Texts: *Mathematics for Elementary Teachers,* Eugene Krause.
Fundamentals of Mathematics I – Group Exercises, C. Ray Rosentrater

Evaluation:

Group exercises	20%
Homework	20%
Expository Paper	5%
Exams (2)	35%
Cumulative Final	20%

Group Exercises: You will be assigned to various groups as the semester progresses. These groups will work on exercises from the supplemental materials package. Responsibility for writing up the submitted copy of the group work will be rotated among the group members. All members of the group should sign the group exercise cover sheet. There are spaces to sign for your work done both in and out of class. Your signature indicates your affirmation that you were present and contributing to the group's effort. *You will receive no credit for group work when you are absent and group work cannot be "made up."*

Make sure that all members of the group are allowed to contribute. In the past when one mathematically strong member has been allowed to dominate a group, the group has not done as well as other groups which lacked a strong member but which worked together.

*Paper***:** This paper provides a forum for you to explore in depth a mathematical idea, development, or philosophy. Typically these papers treat some historical development in mathematics. Your writing is expected to conform to the same high standards of organization, development, and composition that would be expected in a Philosophy or English course. **The paper is due 4:00 pm, Thursday, November 17.** A detailed description of the writing assignment will be posted to the course web site.

Exams: While the exams will include computational problems, much of the exams will be short answer and essay questions. You will be asked to give examples, to evaluate and compare, and to discuss

the implications of the ideas you have studied. The first exam will contain relatively more computational problems with the concentration moving more toward conceptual questions on subsequent exams. You should not expect the exams to consist of problems similar to the homework problems. The exams will seek to assess the degree to which you understand the fundamental definitions and ideas of the course. Detailed descriptions of the exams will be available on the course web site.

Each exam will contain a classic result whose statement and proof you will be expected to reproduce. The three classic results are: Cantor's proof of the unaccountability of the decimal/real numbers, Euclid's proof of the infinitude of prime numbers, and a proof that the square root of 2 is not a rational number.

*Homework***:** Homework will be assigned daily and is due at the beginning of the next regular (non-exam) class meeting. Homework will be posted to the course web site at least three days before it is due. **Late homework will be penalized at a rate of 10% per calendar day**.

Homework papers should be neat, organized, and clearly presented. Prose explanations of your work should be regularly included and the answers to application problems should always be interpreted in terms of the original question using complete English sentences. *The explanation is the most important part of any solution.* Papers with multiple pages should be stapled.

Problems should be logically laid out with appropriate connecting prose and sufficient space so that they are easily readable. Papers not meeting these standards may have the scores reduced or may be returned ungraded at the grader's discretion.

Collaboration on homework is expected and encouraged. There will be no reduction in score due to working with others provided the following guidelines are adhered to:

- All students in the group *understand* the solution and are not merely copying solutions. Each solution should be in that student's own words.
- *All collaboration is credited*. This will generally take the form of a note at the end of a solution like "the solution was compared with that of John Martin for verification" or "this solution was developed in collaboration with Jane Smith and Samantha Jones."

Every homework assignment should include a statement identifying those from whom you have received assistance (including the instructor) or a statement that the work is solely your own. There is no penalty for working with others so long as the collaboration is acknowledged. Papers lacking the credits statement will be penalized 10%.

Class Meeting Format: The first 15 minutes of the class period will be devoted to opening activities, house keeping chores, and questions.

When you have questions you should prepare them carefully before class. If your question is related to a homework problem, you should be ready with a statement of the problem, what you were able to do with the problem and the nature of the difficulty. Spending time searching for the statement of a problem and formulating your question after you have been recognized is discourteous to your peers. The rest of the class is expected to show courtesy as well by listening and offering suggestions.

Questions to track down computational errors are not appropriate for class time and should be asked individually in my office.

The remainder of the period usually will be devoted to group work. If your group is unable to complete the work during class time, you should set a time when you can all get together to complete it. Group work should be turned in either at the end of class or at the beginning of the next class period.

Electronic Devices: Cell phones, iPods and mp3 players should be turned off or set to silent (not vibrate) during class. You are expected to actively engage the course material with your fellow students. This

is not possible if you are talking on your cell, texting in your purse, updating your facebook page, tweeting or checking your email under the table. If you are expecting an important call (your sister is about to give birth or you are awaiting news about the results of your grandmother's surgery), let me know at the beginning of class and we will make allowance for such exceptional circumstances.

Absence: While attendance is expected and absence is unwise, there is no formal penalty for absence. Responsibility is expected. If you are forced to miss class for some reason, you should make arrangements for your homework to be brought to class for you. If you know you will be absent on a particular day or for several days, you should make prior arrangements with me as to when you will submit the homework. **Group work cannot be made up**.

Dishonesty: Dishonesty of any kind will result in loss of credit for the work involved. Major or repeated infractions will result in dismissal from the course with a grade of F. Collaboration is encouraged, but mere copying of another's work is dishonest. Give credit on all collaborative work.

Students with Special Needs: Students who have been diagnosed with a disability (learning, physical or psychological) are strongly encouraged to contact the Disability Services office as early as possible to discuss appropriate accommodations for this course. Formal accommodations will only be granted for students whose disabilities have been verified by the Disability Services office. These accommodations may be necessary to ensure your full participation and the successful completion of this course. Please contact Sheri Noble, Interim Coordinator of Disability Services (x6186, snoble@westmont.edu) as soon as possible.

Schedule of Topics:

Aug.	29	Introduction/problem solving
	31	Logic
Sept.	2	(Cont.)
	5	(Cont.)
	7	Sets
	9	(Cont.)
	12	(Cont.)
	14	(Cont.)
	16	Binary Operations and their properties
	19	(Cont.)
	21	Functions
	23	(Cont.)
	26	Sizes of Sets
	28	(Cont.)
	30	Numbers and Counting
Oct.	3	Intro. to Numeration Systems
	5	Place Value Systems
	7	**Exam 1** (Through Numbers and Counting)
	10	*Fall Holiday*
	12	Foundations of Addition
	14	Foundations of Multiplication
	17	Addition Algorithms
	19	Multiplication Algorithms
	21	Order in the Whole Numbers
	24	Subtraction in the Whole Numbers
	26	Subtraction Algorithms
	28	Division in the Whole Numbers
	31	Introduction to Number Theory
Nov.	2	Infinitude of Primes
	4	Divisibility Tests
	7	GCD's and LCM's
	9	Relations
	11	**Exam 2** (Through Divisibility Tests)
	14	A Concrete Model of the Integers
	16	An Abstract Model for the Integers
	18	(Cont.)
	21	Subtraction and Order in the Integers
	23	*Thanksgiving*
	25	*Break*
	27	A Concrete Model for the Rational Numbers
	30	An Abstract Model for the Rational Numbers
Dec.	2	Algebraic and Order Properties of the Rational Numbers
	5	The Real Numbers
	7	(Cont.)
	9	Review
	13	**Final Exam, 12:00 noon**