MA 165: Fundamentals of Mathematics II (2 Units)

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Office hours: Tues. 1:30 – 3:00
Thurs. 10:30 – 12:00
Fri. 9:30 – 10:25

Prerequisite

Satisfaction of the Westmont College mathematics entrance requirement.
(Note that, while MA 160 is not a formal prerequisite, there will be occasions where vocabulary from MA 160 will be referenced. You may be familiar with the terminology from other courses or group mates should be able to supply the missing ideas. If you are a fairly strong student, this should pose not problem. Students who are less sure of their abilities would do well to complete MA 160 before enrolling in MA 165.)

Course Objectives

The purpose of MA 165 is to promote a thorough understanding of the mathematics you should know in order to be qualified to teach at the elementary level. This includes:

The fundamental ideas underlying probability and expected value.

Basic techniques of communicating results and making conclusions about statistical data.

The Central Limit Theorem and its importance in drawing conclusions from data.

Fundamental concepts and vocabulary of geometry including congruence, similarity, symmetry, and transformations.

Basic construction techniques using straight edge and compass and mira.

The ideas of linear, angle, area and volume measurement.

An appreciation for the structures and patterns underlying mathematics.

This course is not intended to correct deficiencies in basic mathematical background – you have demonstrated your basic competency in arithmetic and algebra by satisfying Westmont's mathematics competency requirement as a prerequisite to this course. Though this course will be sensitive to instructional issues, it is not intended to teach you how to teach – teaching methods will be addressed in courses from the Education Department. Rather, its purpose is to deepen your mathematical understanding. Your goal in this course should be to reconsider from a more advanced perspective the mathematical processes and ideas that you encountered during the early part of your academic career. Just as your reading abilities and appreciation of literature should far exceed the level of 'see spot run' or "Where the Red Fern Grows", so too one's knowledge and understanding of mathematics should extend beyond and be deeper than what one would be required to teach.

This course will examine familiar mathematics from a perspective that will be new to most of you. We will focus on the abstract and theoretical structure of mathematics, emphasizing problem solving, communication, and reasoning. Put another way, we will be exploring and communicating the ideas and reasons behind the mathematical ideas.

Relationship to other expectations

The *Mathematics Framework for California Public Schools (2005)* (http://www.cde.ca.gov/ci/ma/cf/) lists six goals for mathematics students.

1. that they develop fluency in computation;
2. that they learn to communicate mathematical ideas precisely;
3. that they develop logical thinking;
4. that they make connections among mathematical ideas and with other disciplines;
5. that they apply mathematics to everyday life;

6. that they develop an appreciation for the beauty and power of mathematics.

More recently, the National Governors Association Center for Best Practices and the Council of Chief State School Officers have been working on a Common Core State Standards Initiative. The final standards (http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf) were released in June 2010 and since have been adopted by 45 of the 50 states. California adopted the Common Core State Standards August 2, 2010. The standards identify the mathematical content that students should master at each grade level. For example, one of the outcomes for second graders is that they be able to "[e]xplain why addition and subtraction strategies work, using place value and the properties of operations." Third graders should be able to "[c]ompare two fractions with the same numerator or the same denominator by reasoning about their size." Moreover, at every level, students are expected to engage in a set of common mathematical practices.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

As elementary school teachers, you will be responsible for helping students and departments achieve these goals. Thus in addition to knowing the basic algorithms and manipulations of mathematics, you must become competent problem solvers, good communicators and be able to reason mathematically. You will need to see how mathematics builds on itself. If you are to teach your students to appreciate mathematics, you must learn to value mathematics yourself.

The Common Core State Standards use 10 major headings under which the outcomes for each grade level are organized. At most five of the major categories are addressed at any grade level. The major headings and the grade levels at which they are first introduced are listed below.

1. Counting and Cardinality (Grade K)
2. Operations and Algebraic Thinking (Grade K)
3. Number and Operations in Base Ten (Grade K)
4. **Measurement and Data** (Grade K)
5. **Geometry** (Grade K)
6. Number and Operations – Fractions (Grade 3)
7. Ratios and Proportional Relationships (Grade 6)
8. The Number System (Grade 6)
9. Expressions and Equations (Grade 6)
10. **Statistics and Probability** (Grade 6)

MA 165 addresses the three areas in bold while the companion course, MA 160, addresses the remaining seven areas. It is important to understand, however that mathematics, and so this course, does not easily split up into simple categories. Just try doing measurement, probability, statistics, or algebra without understanding numbers. The ideas underlying numbers depend in turn upon patterns, functions, logic, and are often best understood in terms of geometry and measurement. In general, mathematics consists of the search for patterns and the logical analysis of systems and structures. Geometry is meaningless until one begins to understand its patterns and structure via logical reasoning, but there are no patterns or structures to understand without some elementary geometric ideas in the first place.

It is interesting to note that the areas least likely to be named as part of mathematics, namely patterns and logical reasoning, play such a central role in mathematics. Unfortunately, many students come away from mathematics lessons and courses believing mathematics to be an activity where instructors give algorithms and answers and students drill on techniques. While some drill is needed, the driving force in mathematics should always be towards conceptual understanding.

You will find that this course is structured to provide you with the experience of developing your own mathematical ideas. You will work on exercises that will lead you toward understanding of concepts behind familiar as well as new ideas. Your instructor will be available to help you when you when you get stuck and to provide an over-arching context for what you are doing. The important point is that you will be doing rather than listening to lectures.

Conceptual Goals:

Students in Mathematics 165 should be able to reason, solve problems, and communicate effectively in the following areas:

1. The use of descriptive statistics – both graphical and numerical;
2. The idea of inferential statistics;
3. Basic probability calculations and rules;
4. The Central Limit Theorem;
5. Expected value;
6. The geometric concepts of dimension, connectedness, transformation, and symmetry;
7. The ideas of congruence and similarity and their relationship to measurement;
8. The ideas of graph theory;
9. The Pythagorean Theorem;
10. Recognition and communication of mathematical patterns.

Affective (Non-Content) Goals:

Students in Mathematics 165 should:

1. Gain a greater understanding of the nature of mathematics;
2. Appreciate the importance of understanding rather than memorizing;
3. Value cooperative learning and see it modeled;
4. Acquire a positive attitude towards and a lack of fear of mathematics.

Goals and GE:

MA 165 satisfies the GE skills requirement *Quantitative and Analytical Reasoning*. There are two general types of mathematical models of the world. Statistical models analyze observed data to provide descriptions of physical and social phenomena. These models can then be used to make predictions and draw conclusions about the modeled situation. The second type of model begins with general principles and uses algebraic models to predict the behavior of a system. MA 165 develops models that fall into the first category. We will spend several weeks studying various types of statistical descriptors and discussing ways in which the characteristics of data can be communicated accurately and effectively in both numerical and graphical forms. By studying the properties of the standard summary statistics and the properties of graphical representations of data, you will become better equipped to recognize and prevent manipulative uses of statistics.

Links to Institutional Learning Outcomes:

Critical Thinking: While operating within the context of mathematical thinking, we will see how a single phenomenon can be viewed, analyzed, and presented using multiple conceptual models. Moreover, you will begin to understand the limits of the various forms of data analysis and presentation. Throughout the course you will be challenged to clearly explain not just what is true, but why it is true.

Diversity: While studying the Pythagorean Theorem, we will study proofs contributed by multiple cultures. We will see how the understandings and predispositions of the cultures influenced their particular contributions.

Written and Oral Communication: Particularly when we are studying geometric constructions, you will be challenged to provide clear and precise instructions for how the constructions are performed and why they

work. In addition, you will sharpen your research and expository skills be preparing a paper that develops or extends and idea we have studied in class.

Research and Technology: In the process of writing up your expository paper, you will have another opportunity to hone your skills in appropriate documentation of your sources. While learning about statistics, you will be introduced to basic spreadsheet manipulations.

Texts: *Mathematics for Elementary Teachers,* Eugene Krause.
Group Exercises in Probability, Statistics, and Geometry, Ray Rosentrater

Materials: Graph paper, compass, ruler, and scissors.

Evaluations:

Group exercises	15%
Homework	20%
Paper (Due March 23)	10%
Exams (2)	35%
Cumulative Final	20%

Group Exercises: You will be assigned to various groups as the semester progresses. These groups will work on exercises from the supplemental materials package. Exercise sets that are not completed during the class meeting time should be completed outside of class and turned in by the next class meeting. Responsibility for writing up the submitted copy of the group work will be rotated among the group members. All contributing members of the group should sign the cover sheet. There are spaces to sign for your work both in and out of class. Your signature indicates your affirmation that you were present and contributing to the group's effort. You will receive no credit for group work when you are absent and *group work cannot be "made up."*

Homework: Weekly homework is due at the beginning of the Monday class meeting. In cases where class does not meet on Monday, the homework is due the following Wednesday. A list of assignments with their due dates will be posted on the course web site.

Homework papers should be neat, organized, and clearly presented. Prose explanations of your work should be regularly included and the answers to application problems should always be interpreted in terms of the original question using complete English sentences. *The explanation is the most important part of any solution.* Papers with multiple pages should be stapled.

There should be no need for arrows on the page directing the reader's attention from one section to another. Rather, problems should be logically laid out with appropriate connecting prose and sufficient space so that they are easily readable. Papers not meeting these standards may have the scores reduced or may be returned ungraded at the grader's discretion.

Collaboration on homework is expected and encouraged. There will be no reduction in score due to working with others provided the following guidelines are adhered to:

- All students in the group *understand* the solution and are not merely copying solutions.
- *All collaboration is credited*. This will generally take the form of a note at the end of a solution like "the solutions were compared with that of John Martin for verification" or "this solution was developed in collaboration with Jane Smith and Samantha Jones."

All papers should include either a list of credits (who you worked with) or a statement that the work is solely your own.

Mere copying of another's work is dishonest and unacceptable.

*Paper (Due March 23)***:** This paper provides a forum for you to explore in depth a mathematical idea, development, or philosophy. Typically these papers treat some historical development in mathematics. Your writing is expected to conform to the same high standards of organization, development, and composition that would be expected in a Philosophy or English course. A

detailed description of the writing assignment will be posted on the course web site.

It is very important that you give careful attention to the process of giving credit to your sources. The body of the paper must include citations not only for quotations but also for ideas, facts, lines of thought and illustrations. Be sure to visit the college's web page on plagiarism and to understand how to properly credit the material you read in the process of preparing your paper. Failure to properly credit sources has very severe consequences.

Exams: While the exams will include some computational problems, the majority of the exams will be short answer and essay questions. You will be asked to give examples, to evaluate and compare, and to discuss the implications of the ideas you have studied. You should not expect the exams to consist of problems similar to the homework problems. The exams will seek to assess the degree to which you understand the fundamental definitions and ideas of the course.

Each exam will contain a classic theorem. The two classic results are: The Central Limit Theorem and the Pythagorean Theorem. You will be expected to state and explain the Central Limit Theorem and to give both a statement and a complete proof of the Pythagorean Theorem.

The **final exam** will be Wednesday, May 2 at 12:00 noon. Exceptions can be made only by petition to the registrar and are rarely granted.

Absence: While attendance is expected and absence is unwise, there is no formal penalty for absence other than the fact that you will not receive credit for the group work on the days you are absent.

Responsibility is expected. If you are forced to miss class for some reason, you should make arrangements for your homework to be brought to class for you. Your absence is not an acceptable reason for your homework's absence. You should also make arrangements to get notes from the day's work. Office hours are for those who have questions about what was done in class or on the homework. They are not intended as replacements for class attendance.

Dishonesty: Dishonesty of any kind will result in loss of credit for the work involved. Plagiarism in the expository paper will also result in a report being filed with the provost's office. Major or repeated infractions will result in dismissal from the course with a grade of F. Collaboration is encouraged, but mere copying of another's work is dishonest. Give credit on all collaborative work including anyone with whom you checked answers.

Schedule:

January	9	Intro to Probability
	11	Probabilities of Events
	16	*Monday Schedule -- No class*
	18	Probabilities of Compound Events
	23	Expected Value
	25	Descriptive Statistics: Graphical Methods
	30	Descriptive Statistics: Numerical Methods
February	1	Inferential Statistics
	6	Central Limit Theorem
	8	Introduction to Graph Theory
	13	Basic Geometric Notions
	15	**Exam I** (Probability and Statistics)
	20	**President's Holiday**
	22	Geometric Objects
	27	Geometric Constructions: The tools
	29	Using Geometric Constructions
March	5	Introduction to Transformations

	7	More Transformations
	12	**Spring**
	14	**Vacation**
	19	Isometries
	21	More Isometries
	26	Specifying an Isometry
	28	All about Isometries
April	2	Symmetry
	4	Finite Groups of Symmetries
	9	***Easter Recess***
	11	Basics of Measurement
	16	**Exam II** (Graph theory through finite symmetry groups)
	18	Unified Systems of Measurement
	23	Pythagorean Theorem
	25	(Cont.)
May	2	12:00 - 2:00 **FINAL EXAM**