

Calculus I

Name: \_\_\_\_\_

Fall 2007

Instructions: You have 65 minutes for this exam. The exam is closed book, closed notes. A calculator is allowed, but you must show all of your work. **Your work is your answer.** If you have any questions, please ask immediately! Good luck.

## PRACTICE EXAM

**Problem #1:**

Find the derivative of the following ( $A$  is a constant):

a)  $f(x) = \frac{A^2}{x^7} - A \sqrt[16]{x - A}$

b)  $f(x) = x^e - Ae^x + \ln(A^5x) - \sin(Ax)$

**Problem #2:**

Find all antiderivatives of  $g(t) = t^{1/5} - \frac{1}{\sqrt{2}t^3} + \cos(t)$

**Problem #3:**

Check that  $y = -\frac{1}{t+C}$  is a solution to the differential equation:

$$y' = y^2.$$

**Problem #4:**

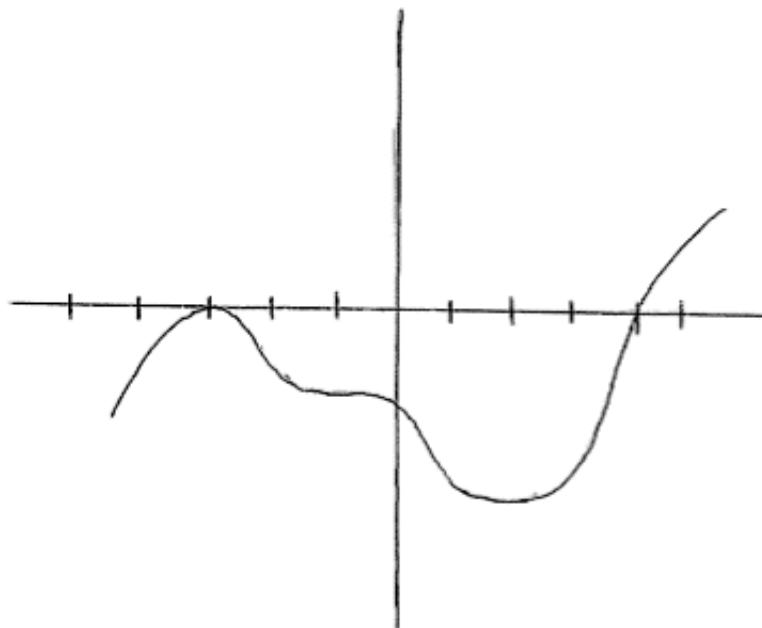
Let  $h(x) = e^x - \ln(x)$ .

a) Find the equation of the line through the points  $(3 - d, h(3 - d))$  and  $(3, h(3))$ .

b) Carefully explain how you could use your answer from part a) to calculate  $h'(3)$ . Your answer should use a diagram to help your explanation.

c) Find the equation of the line tangent to  $h(x)$  at  $x = e$ . (You should not use your work from parts a) and b).)

For Problems #5 and #6 use the graph of the function  $f(x)$  below.



**Problem #5:**

Sketch (on a new set of axes) the graph of the **derivative** for  $f(x)$ .

**Problem #6:**

Sketch (on a new set of axes) the graph of a possible antiderivative for  $f(x)$ .

**Problem #7:**

Consider  $f(x) = \frac{1}{\sqrt{x-2}}$ . Find  $f'(x)$  **using the definition of the derivative**. No credit will be given for any other method of obtaining the answer.

**Problem #8:**

Find the point on the graph of  $y = \frac{1}{x} + x$  which is closest to the origin.

(Hint: the distance between two points is  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ , but you may minimize the **square** of the distance.)

**Problem #9:**

Gilda Smith is a goldsmith in Gaul who gilds goblets. She is going to gild the outside of a triangular goblet with volume  $200 \text{ cm}^3$ . The ends of the goblet are equilateral triangles with sides of length  $s$  cm (one of the ends is open). The goblet is  $w$  cm tall. She is galled to discover that the goldsmiths' guild specifies that the gold for the triangular bottom of the goblet costs 6 guilders per  $\text{cm}^2$  and the gold for the sides costs 10 guilders per  $\text{cm}^2$ . What length  $s$  will minimize the total cost for the gold?

The area of an equilateral triangle with sides of length  $s$  is  $\frac{\sqrt{3}}{2}s^2$ . The volume of the goblet described is  $\frac{\sqrt{3}}{2}s^2w$ .

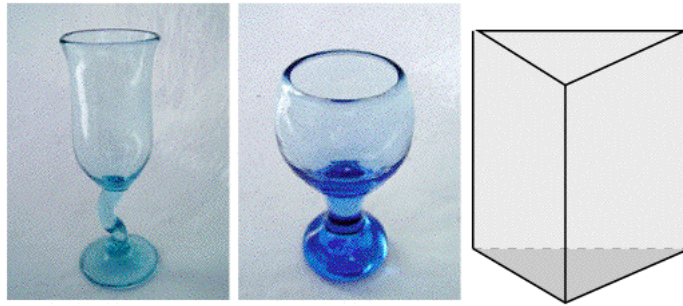


FIGURE 1. Some of Gilda's goblets