Math 9: Problem from 10/1/07 class

Find all extrema of $f(x) = \sin(2x) - \cos(2x)$**.**

Step 1: Find the derivative:

$$f'(x) = 2\cos(2x) + 2\sin(2x).$$

Step 2: Find the stationary points:

$$\begin{aligned}
2\cos(2x) + 2\sin(2x) &= 0 \\
\cos(2x) + \sin(2x) &= 0 \\
\sin(2x) &= -\cos(2x) \\
& \quad \text{since } \cos(2x) \neq 0 \\
\tfrac{\sin(2x)}{\cos(2x)} &= -1 \\
\tan(2x) &= -1 \\
2x &= -\tfrac{\pi}{4} + \pi c \\
x &= -\tfrac{\pi}{8} + \tfrac{\pi}{2} c
\end{aligned}$$

where $c = \ldots, -2, -1, 0, 1, 2, \ldots$.

Step 3: Find out if they are maxima or minima:

$f''(x) = -4\sin(2x) + 4\cos(2x)$.

The period of the functions $f(x)$ and $f''(x)$ is $2\pi/2 = \pi$ since both $\cos x$ and $\sin x$ have period 2π. So we only need to check the stationary points in the interval $[-\pi/2, \pi/2]$:

$$x = -\frac{\pi}{8}, \frac{3\pi}{8}$$

To do this plug into $f''(x)$:

$$f''(-\pi/8) = -4\sin(-\pi/4) + 4\cos(-\pi/4) = -4(-\sqrt{2}/2) + 4(\sqrt{2}/2).$$

Since this is positive, $x = -\pi/8$ is a minimum. The function $f(x)$ has period π, so in fact all the numbers $x = -\pi/8 + \pi c$ for $c = \ldots, -1, 0, 1, \ldots$ are minima.

Similarly,

$$f''(\pi/2) = -4\sin(3\pi/4) + 4\cos(3\pi/4) = -4(\sqrt{2}/2) + 4(-\sqrt{2}/2)$$

Since this is negative, $x = 3\pi/8$ is a maximum. The function $f(x)$ has period π, so in fact all the numbers $x = 3\pi/8 + \pi c$ are maxima.