Project 2

Carefully answer the following questions on a separate sheet of paper. Work with a partner if you like.

1. Give examples of functions $f(x)$ and $g(x)$ for which $h(x) = f(x)g(x)$ has a derivative which is **not** $f'(x)g'(x)$.

2. Give examples of functions $f(x)$ and $g(x)$ for which $k(x) = f(x)/g(x)$ has a derivative which is **not** $f'(x)/g'(x)$.

3. Consider the linear functions $f(x) = 3x - 1$ and $g(x) = -2x + 5$. What is the derivative of $h(x) = f(x)g(x)$?

4a. The product of two numbers a and b can be represented by the area of a box with sides of length a and b. Suppose that $f(x)$ and $g(x)$ are both increasing functions. Sketch a picture of a box with sides of length $f(x + \Delta x)$ and $g(x + \Delta x)$. Subdivide the box by drawing a box inside it with sides of length $f(x)$ and $g(x)$. (x is a fixed number in this example.)

4b. Let $h(x) = f(x)g(x)$. Use the picture you've drawn to write down an expression for the quantity $h(x + \Delta x) - h(x)$. The most convenient answer will represent the area of a vertical box plus the area of a horizontal box.

4c. Use the answer from 4b to write down an expression for $h'(x)$ using the definition of derivative.

4d. Use your answer from 4c. and properties of limits to prove that:

$$h'(x) = f'(x)g(x) + g'(x)f(x).$$

5. Use the product rule (the equation in 4d) to confirm your answer to question 3.