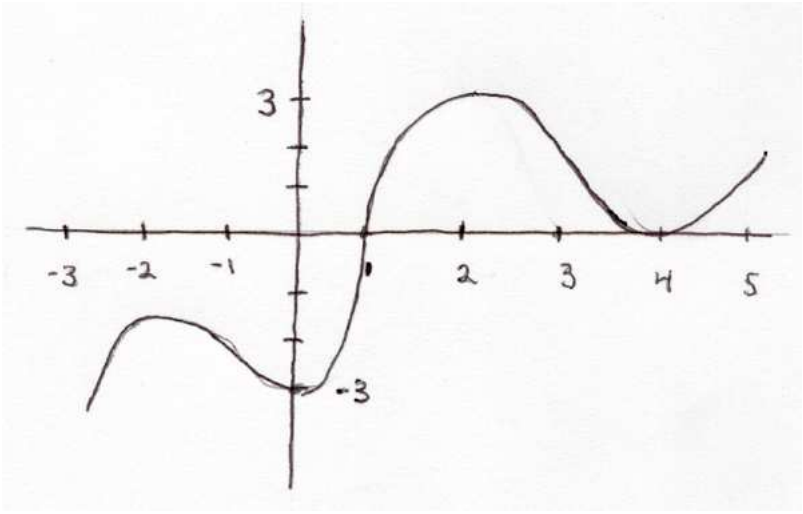
Calculus 1: Practice Exam 1 Solutions

Name:

(1) Explain in words why the derivative of the linear function $f(x) = mx + b$ (m and b are fixed real numbers) is $f'(x) = m$.

Answer: The graph of $f(x)$ is a straight line. At any point $x = a$, the tangent line is just the line itself. The slope of the tangent line is the derivative, so the slope of the tangent line when $x = a$ is just the slope of $f(x)$ which is m. Thus, $f'(x) = m$ for any number x.

(2) Here is a graph of $f(x)$.



(a) Draw a possible graph for $f'(x)$ (on a new set of axes). Be as accurate as possible.

We'll go over this in class

(b) Draw the graph of an antiderivative of $f(x)$ (on a new set of axes). Be as accurate as possible.

We'll go over this in class

(3) Suppose that for some function $m(x)$, $m'(-3) = 0$ and $m''(-3) = -\frac{2}{5}$. What can you say about the graph of $m(x)$ at $x = -3$.

Explain.

Answer: The graph of $g(x)$ has a maximum at $x = -3$. It has a stationary point since $m'(-3) = 0$ and it has a maximum since the graph is concave down there.

(4) Explain graphically (in terms of secant lines and tangent lines) why the definition of the derivative of $f(x)$ at $x = a$ is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Answer:

(5) The following limit expresses the derivative of a certain function $h(x)$ at a certain point a. What are the function and the point?

$$\lim_{h \to 0} \frac{2^3 2^h - 8}{h}$$

Answer: The function is $h(x) = 2^x$ and the derivative is taken at the point $a = 3$.

(6) Suppose that k and r are fixed real numbers. Let

$$f(x) = kx^5 - \frac{r}{\sqrt[5]{x^3}} + \sqrt[3]{x}^2$$

Show and explain all your work.

(a) What is $f'(x)$?

Answer:

$$5kx^4 + r\frac{3}{5}x^{-\frac{8}{5}} + \frac{2}{3}x^{-\frac{1}{3}}$$

Using the power rule. $x^{\frac{-8}{5}}$ can be rewritten $\frac{1}{\sqrt[5]{x^8}}$.

(b) Find all the antiderivatives for $f(x)$.

Answer: Using the power rule for antiderivatives, we see that the antiderivatives of $f(x)$ are:

$$\frac{1}{6}(kx^6) - r(\frac{5}{2})x^{\frac{2}{5}} + \frac{3}{5}x^{\frac{5}{3}}$$

(7) Use the definition of the derivative to find $f'(x)$ if $f(x) = \frac{2}{\sqrt{x-3}}$.

Answer:

$$f'(x) = \lim_{h\to 0} \frac{\frac{2}{\sqrt{(x+h-3)}} - \frac{2}{\sqrt{x-3}}}{h}$$

$$= \lim_{h\to 0} 2 \cdot \frac{\frac{\sqrt{x-3}-\sqrt{x+h-3}}{\sqrt{x-3}\sqrt{x+h-3}}}{h}$$

$$= 2 \cdot \lim_{h\to 0} \frac{\sqrt{x-3} - \sqrt{x+h-3}}{h\sqrt{x-3}\sqrt{x-3+h}}$$

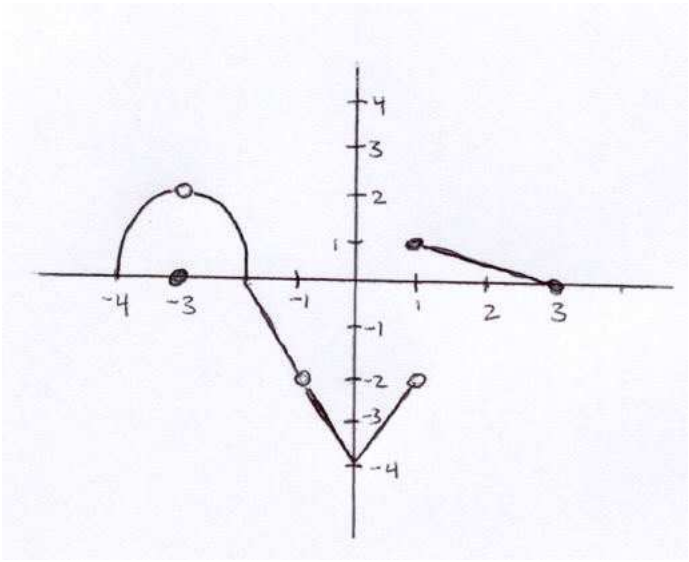
$$= 2 \cdot \lim_{h\to 0} \frac{\sqrt{x-3} - \sqrt{x+h-3}}{h\sqrt{x-3}\sqrt{x-3+h}} \cdot \frac{\sqrt{x-3} + \sqrt{x-3+h}}{\sqrt{x-3} + \sqrt{x-3+h}}$$

$$= 2 \cdot \lim_{h\to 0} \frac{-h}{h\sqrt{x-3}x - 3 + h(\sqrt{x-3} + \sqrt{x-3+h})}$$

$$= 2 \cdot \lim_{h\to 0} \frac{-1}{\sqrt{x-3}x - 3 + h(\sqrt{x-3} + \sqrt{x-3+h})}$$

$$= \frac{-2}{(x-3)(2\sqrt{x-3})}$$

(8) Here is the graph of a certain function $h(x)$. Find the following limits if they exist. If they do not exist explain why.



(a) $\lim_{x\to -3} h(x)$

(b) $\lim_{x\to -1} h(x)$

(c) $\lim_{x\to 0} h(x)$

(d) $\lim_{x\to 1} h(x)$

(e) $\lim_{x\to 1^+} h(x)$

(f) $\lim_{x\to 1^-} h(x)$

(g) $\lim_{x\to 3^-} h(x)$

(h) Is $h(x)$ continuous at $x = -3$? Why or why not?

(9) A region is going to be fenced in with a rectangular fence and then more fence will cut the region into two corrals of equal size. If the fence costs \$25 per foot what dimensions for the sides will maximize area if the total cost is to be \$1000?

Answer: We get the equations: $A = lw$ and $25(2l + 3w) = 1000$. Solving the second equation for l we find: $l = 20 - \frac{3}{2}w$. Plugging into the area equation we obtain:

$$A = 20w - \frac{3}{2}w^2$$

.

To maximize this we take the derivative and set it equal to zero to obtain:

$$20 - 3w = 0 \Longrightarrow w = \frac{20}{3}$$

This means that $l = 10$.

(10) Find two solutions to the differential equation:

$$f''(x) = 2f(x)$$

Answer: Here are two possible answers. Can you find more?

$$f(x) = 3e^{\sqrt{2}x}$$

$$f(x) = -e^{\sqrt{2}x}$$

(11) Suppose that a is a constant. What is the derivative of $g(x) = e^{-ax}$?

Answer: $g'(x) = (-a)e^{-ax}$