Calc 1: Practice Exam 2

Name:

(1) Find derivatives of the following functions:

(a)

$$y = \frac{x^5 + 2x + 1}{\sqrt[3]{x}\cos(x)}$$

(b)

$$y = e^{x^3}\tan(x^4 + 1)$$

(c)

$$y = \cos(e^{x^2})\sin(e^{-x^2})$$

(d)

$$y = \frac{\tan(\sqrt{x} - \sqrt[3]{(x)})}{x^2 e^{5x}}$$

(2) Consider the curve defined by the equation

$$ye^x + xe^y = e^{xy}$$

(a) Find $\frac{dy}{dx}$

(b) Find the equation of the tangent line to the curve at the point (1,0).

(3) Show all the steps for finding the derivative of:

(a) $y = \arcsin(x)$

(b) $y = \arccos(x)$

(c) $y = \arctan(x)$

(4) Suppose that $f(t)$ is a solution of the DE $y' = t^2y + t$ and that $f(1) = 2$. Find the equation of the tangent line to $f(t)$ at the point (1,2).

(5) Suppose that $h(t)$ is a solution of the DE $y' = e^t y$ and that $h(-2) = 1$. Find an equation of the line tangent to h at the point $(-2, 1)$.

(6) Find the following limits. Explain all your steps.

(a)

$$\lim_{x\to\infty} \frac{\ln(x)}{\sqrt[3]{x}}$$

(b)

$$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$$

(c)

$$\lim_{x\to\infty} \frac{x^2 - 1}{2x^2 + 1}$$

(7) Find the point on the line $y = -3x + 2$ which is closest to the origin.

(8) When a person coughs, the velocity of the air stream is related to the radius of the trachea by:

$$v(r) = k(r_0 - r)r^2$$

The normal radius of the trachea is r_0 and k is a constant. The radius r of the trachea is restricted so that $\frac{1}{2}r_0 \leq r \leq r_0$ in order to prevent suffocation.

Find the value of r which makes v an absolute maximum.

(9) Find the speed of the following curve at the point when $t = 0$:

$$\begin{cases} x(t) &= 2\sin(4t) \\ y(t) &= 3\sin(5t) \end{cases}$$

(10) If two resistors with resistances R_1 and R_2 are placed in parallel in a circuit, the total resistance of the circuit, R is related to R_1 and R_2 be the equation:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R_1 is increasing at a rate of .4 ohms per second and R_2 is increasing at a rate of .6 ohms per second, how fast is R changing when $R_1 = 20$ ohms and $R_2 = 30$ ohms?

(11) A baseball diamond is a square with sides 90 ft. A batter hits the ball and runs toward first base with a speed of 24 feet per second. At what rate is his distance from 3rd base increasing, when he is halfway to 1st base? (The bases are located at the corners of the square).

(12) Explain why the function $y = 2000x^{91} - 24x^2 + 15$ must have at least one root. (Your answer should involve a theorem discussed in class.)

(13) When I wake up in the morning it is 45° F. At lunch time it is 68° F. Explain how I know that there was some time in the morning when it was exactly 15π°F.

(14) Let $f(t) = t^3 + t + 3$. Explain why there is some number c between -1 and 1 such that $f'(c) = 2$. Your answer should use a theorem we discussed in class.