Math 10: Practice Exam 1

Name:

You may use a calculator on the exam and a 3"×5" notecard with notes on it. You have 65 minutes for the exam. Show all of your work. Your work **is** your answer. The practice exam is longer and more difficult than the actual exam.

Problem 1: Consider the definite integral $I = \int_e^{4e} \ln(x^2 + x)\ dx$.

(a) What is the Left Hand approximation to I with $n = 3$?

(b) Calculate the first derivative of $f(x) = \ln(x^2 + x)$ and use this to show that L_3 is an underestimate of I.

(c) What is an upperbound on $|I - L_3|$?

(d) What value of n would be needed in order to assure that $|I - L_n| \leq .001$?

(e) Calculate the midpoint approximation to I with $n = 3$.

(f) What is an upperbound on $|I - M_3|$?

Problem 2: Let $f(x)$ be any continuous function on the interval $[2, 3]$ such that $f'(x) \leq 18$. Show that $f(x) \leq 18(x-2) + f(2)$.

Problem 3: Consider the initial value problem:

$$y' = y^2 \qquad y(0) = 1$$

(a) Use Euler's method with four steps of size 0.2 to estimate $y(0.8)$.

(b) Show that the exact solution of the IVP is $y(t) = \frac{1}{1-t}$.

Problem 4: Compute the area contained in the region bounded by:

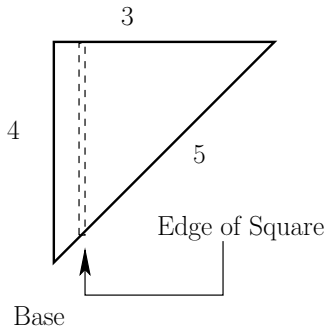
$$\begin{aligned} x &= 1 \\ x &= 4 \\ y &= -(x-3)^2 + 4 \\ y &= \ln(x) \end{aligned}$$

Problem 5: Write down an integral representing the arclength of the curve

$$y = \arcsin(x)$$

from the point $(0, 0)$ to the point $(1, \pi/2)$. You should actually work out the derivative. You do not need to solve the integral.

Problem 6: Compute the volume of an object which has a base that is a right triangle with sides of length 3, 4, and 5 as indicated in the figure. The cross-sections are squares. See the figure.



Problem 7: Compute the volume of the object obtained by rotating the region bounded by

$$\begin{aligned} y &= \sin(x) \\ y &= 0 \\ x &= 0 \\ x &= \pi \end{aligned}$$

around the x-axis. (Hint: Use the trig identity: $\cos(2x) = 1 - 2\sin^2(x)$.)

Problem 8: Compute the volume of the object obtained by rotating the region bounded by:

$$\begin{aligned} y &= \sin(x^2) \\ y &= 0 \\ x &= 0 \\ x &= 1 \end{aligned}$$

around the y axis.

Problem 9: A 10 meter rope which weighs 2 kg/m is hanging off the top of a 10 meter building. How much work is done in hauling the rope to the top of the building? (The acceleration due to gravity is $g = -9.8$ m/s^2.)

Problem 10: Find all solutions to the DE: $y' = t(y-1)$.

Problem 10: Integrate the following:

(a)
$$\int x^2 e^x \, dx$$

(b)
$$\int x e^{x^2} \, dx$$

(c)
$$\int_0^1 x \sin x \, dx$$

(d)
$$\int (\sin x) e^x \, dx$$

(e)

$$\int_1^2 \frac{5x+2}{(x+4)(2x-1)}\,dx$$

(f)

$$\int \frac{2x^3+7x^2+44x+58}{(x+3)^2(x^2+4)}$$