Math 19: Practice Exam 1

Name:

You may use a non-graphing calculator on the exam and a notecard with notes on it. You have 65 minutes for the exam. Show all of your work. Your work **is** your answer.

(1) Determine the angle between the following vectors. You may leave your answer in non-decimal form; your answer may have an inverse trig function in it.

$$\begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} \sqrt{3} \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

(2) For what value of b are the vectors $\mathbf{v} = \begin{bmatrix} -2 \\ b \\ 8 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} b \\ 2b^2 \\ -b \end{bmatrix}$ orthogonal with $\mathbf{w} \neq \mathbf{0}$?

(3) Write the equation of the vector projection of $(2, 1, -2)$ onto $(0, 3, 1)$.

(4) Find the area of the parallelogram determined by $(1, -1, -1)$ and $(6, \pi, -2)$.

(5) Determine if the lines

$$t \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + (1-t) \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix}$$

and

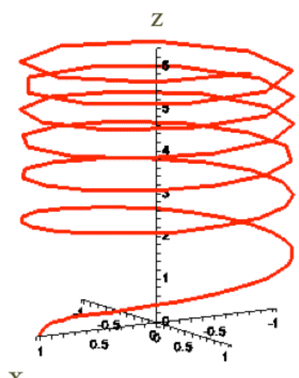
$$s \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + (1-s) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

intersect. If they do, specify the t and s values where they intersect.

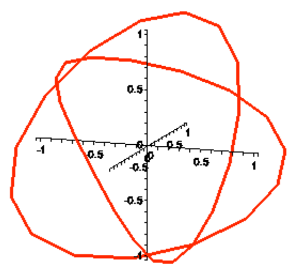
(6) Find a normal vector to the plane $2x - 3y + 4z = 6$.

(7) What is the equation in spherical coordinates for the cylinder given in rectangular coordinates by $x^2 + y^2 = 4$?

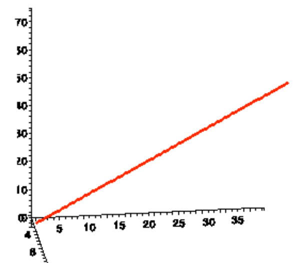
(8) Match each of the following space curves to their equation. Give a brief reason for each choice. Notice that one equation will go unused.



Curve A



Curve B



Curve C

Possible Equations:

$$\mathbf{r}(t) = (\cos(3t), \sin(2t), \cos(2t))$$

$$\mathbf{r}(t) = (e^t, e^{-t}, t^2)$$

$$\mathbf{r}(t) = (\cos(t^2), \sin(t^2), t)$$

$$\mathbf{r}(t) = (t, 2t+1, 4t-2)$$

(9) Find the length of the curve $\mathbf{r}(t) = (\sin(t), \cos(t), \ln|\sec(t)|)$ from $t = 0$ to $t = \pi/4$.

It might be helpful to remember the following formulas:

$$\begin{aligned} \tfrac{d}{dt}(\ln|\sec t|) &= \tan(t) \\ \tan^2(t) + 1 &= \sec^2(t) \\ \textstyle\int \sec(t)dt &= \ln|\sec(t) + \tan(t)| + C \end{aligned}$$

For the following 4 questions, consider the curve:

$$\mathbf{r}(t) = \begin{bmatrix} e^t \\ \cos(e^t) \\ \sin(e^t) \end{bmatrix}$$

(10) Find the unit tangent vector at time t.

(11) Find the unit normal vector at time t.

(12) Find the binormal vector at time t.

(13) Find the curvature at time t.

(14) Use level curves to describe the graph of the surface $f(x, y) = xy$.

(15) Show that $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ does not exist.

(16) Find all first and second partial derivatives of:

$$f(x,y) = e^{-x}\sin(y)$$

On the exam, you will be asked to prove one of the following statements:

(1) The length of the vector $\mathbf{v}$ projected onto $\mathbf{w}$ is:

$$\frac{|\mathbf{w} \cdot \mathbf{v}|}{|\mathbf{w}|}$$

(2)

$$\frac{d}{dt}(\mathbf{u}(t) \cdot \mathbf{v}(t)) = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)$$

(3) Prove or provide a counterexample:

$$||\mathbf{v} \cdot \mathbf{w}|| = ||\mathbf{v}|| \cdot ||\mathbf{w}||$$

(4) If $\mathbf{r}(t)$ is a vector valued function such that $||\mathbf{r}(t)|| = 2$ for all values of t then $\mathbf{r}'(t)$ and $\mathbf{r}(t)$ are orthogonal for all values of t.

(5) Recall that $\kappa(t) = ||\frac{d\mathbf{T}}{ds}||$. Use this and the chain rule to prove that $\kappa(t) = \frac{||\mathbf{T}'(t)||}{||\mathbf{r}'(t)||}$ where $\mathbf{T}$ is the unit tangent vector of $\mathbf{r}(t)$ and $s(t)$ is the arclength of $\mathbf{r}(t)$.

(6) Prove that $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2} = 0$.