Math 19: Practice Exam 1 Partial Solutions

Name:

These are partial answers. In particular, no work is shown. This is designed so that you can check your work–not to provide a model for the sorts of answers I expect.

(1) Determine the angle between the following vectors. You may leave your answer in non-decimal form; your answer may have an inverse trig function in it.

$$\begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} \sqrt{3} \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

Answer:

$$\theta = \cos^{-1}(\frac{1}{3\sqrt{2}})$$

(2) For what value of b are the vectors $\mathbf{v} = \begin{bmatrix} -2 \\ b \\ 8 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} b \\ 2b^2 \\ -b \end{bmatrix}$ orthogonal with $\mathbf{w} \neq \mathbf{0}$?

Answer:

$$b = \pm\sqrt{5}$$

(3) Write the equation of the vector projection of $(2, 1, -2)$ onto $(0, 3, 1)$.

Answer:

$$\frac{1}{10}\begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$$

(4) Find the area of the parallelogram determined by $(1, -1, -1)$ and $(6, \pi, -2)$.

Answer:

$$\sqrt{(2+\pi)^2 + 16 + (6+\pi)^2}$$

(5) Determine if the lines

$$t\begin{bmatrix}0\\1\\2\end{bmatrix} + (1-t)\begin{bmatrix}4\\0\\-1\end{bmatrix}$$

and

$$s\begin{bmatrix}1\\2\\0\end{bmatrix} + (1-s)\begin{bmatrix}0\\0\\1\end{bmatrix}$$

intersect. If they do, specify the t and s values where they intersect.

Answer: They do not intersect.

(6) Find a normal vector to the plane $2x - 3y + 4z = 6$.

Answer: The vector $(2, -3, 4)$ is a normal vector as is any multiple of it.

(7) What is the equation in spherical coordinates for the cylinder given in rectangular coordinates by $x^2 + y^2 = 4$?

Answer:

$$\rho \sin \phi = 2$$

(8) Match each of the following space curves to their equation. Give a brief reason for each choice. Notice that one equation will go unused.

$$\begin{aligned}
\textbf{B}: \quad & \mathbf{r}(t) &=& \quad (\cos(3t), \sin(2t), \cos(2t)) \\
\textbf{unused}: \quad & \mathbf{r}(t) &=& \quad (e^t, e^{-t}, t^2) \\
\textbf{A}: \quad & \mathbf{r}(t) &=& \quad (\cos(t^2), \sin(t^2), t) \\
\textbf{C}: \quad & \mathbf{r}(t) &=& \quad (t, 2t+1, 4t-2)
\end{aligned}$$

(9) Find the length of the curve $\mathbf{r}(t) = (\sin(t), \cos(t), \ln|\sec(t)|)$ from $t = 0$ to $t = \pi/4$.

It might be helpful to remember the following formulas:

$$\begin{aligned}
\frac{d}{dt}(\ln|\sec t|) &= \tan(t) \\
\tan^2(t) + 1 &= \sec^2(t) \\
\int \sec(t)dt &= \ln|\sec(t) + \tan(t)| + C
\end{aligned}$$

Answer: The length is $\ln|\sqrt{2} + 1|$.

For the following 4 questions, consider the curve:

$$\mathbf{r}(t) = \begin{bmatrix} e^t \\ \cos(e^t) \\ \sin(e^t) \end{bmatrix}$$

(10) Find the unit tangent vector at time t.

Answer:

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -\sin(e^t) \\ \cos(e^t) \end{pmatrix}$$

(11) Find the unit normal vector at time t.

Answer:

$$\begin{pmatrix} 0 \\ -\cos(e^t) \\ -\sin(e^t) \end{pmatrix}$$

(12) Find the binormal vector at time t.

Answer:

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \sin(e^t) \\ -\cos(e^t) \end{pmatrix}$$

(13) Find the curvature at time t.

Answer: $\frac{1}{2}$.

(14) Use level curves to describe the graph of the surface $f(x,y) = xy$.

Answer: You should be able to draw level curves.

(15) Show that $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ does not exist.

Answer: The limit as $(0,y) \to (0,0)$ is 0 but the limit as $(x,x) \to (0,0)$ is $\frac{1}{2}$.

(16) Find all first and second partial derivatives of:

$$f(x,y) = e^{-x}\sin(y)$$

Answer: Here they are:

$$\begin{aligned}
f_x(x,y) &= -e^{-x}\sin y \\
f_y(x,y) &= e^{-x}\cos y \\
f_{xy}(x,y) &= -e^{-x}\cos y \\
f_{yx}(x,y) &= -e^{-x}\cos y \\
f_{xx}(x,y) &= e^{-x}\sin y \\
f_{yy}(x,y) &= -e^{-x}\sin y
\end{aligned}$$