Math 19: Practice Exam 2

Name:

You are allowed a $3'' \times 5''$ notecard for the exam. You may use a calculator, but not its graphing or calculus features.

(1) Show (using the limit definition of the derivative) that the function $f(x, y) = x(y-1)^2$ is differentiable at the point $(0, 0)$.

(2) How quickly does the function $f(x, y) = \ln(x^2 + y^2)$ increase or decrease at (1,1) in the direction $\mathbf{u} = \frac{1}{\sqrt{20}}(-4, 2)$.

(3) Find all critical points in $[0, 2\pi] \times [-4, 4]$ of the function:

$$f(x, y) = \sin(x) + \frac{1}{3}y^3 - 4y + \cos(x)$$

and determine whether each is a maximum, minimum, or saddle.

(4) Let $f(x, y) = x^3 y$. Approximate the volume contained between the graph of $f(x, y)$ and the rectangle $R = [0, 3] \times [0, 3]$. Use $n = 3$ subintervals of each interval. Use the upper left hand corner of each subrectangle as the x_{ij}^*.

(5) Let $f(x,y) = \frac{xy}{x^2+y^2}$. Let R be the region in the xy plane between the curves $y = x$, $y = 3x$, $x = 0.5$, and $x = 3$. Compute $\int\int_R f(x,y)dA$.

You may wish to use:

$$\int \ln u du = u \ln u - u + C$$

(6) Use polar coordinates to compute:

$$\int\int_D \sin\left(\frac{x^2+y^2}{2}\right)\ dxdy$$

D is the region pictured below:

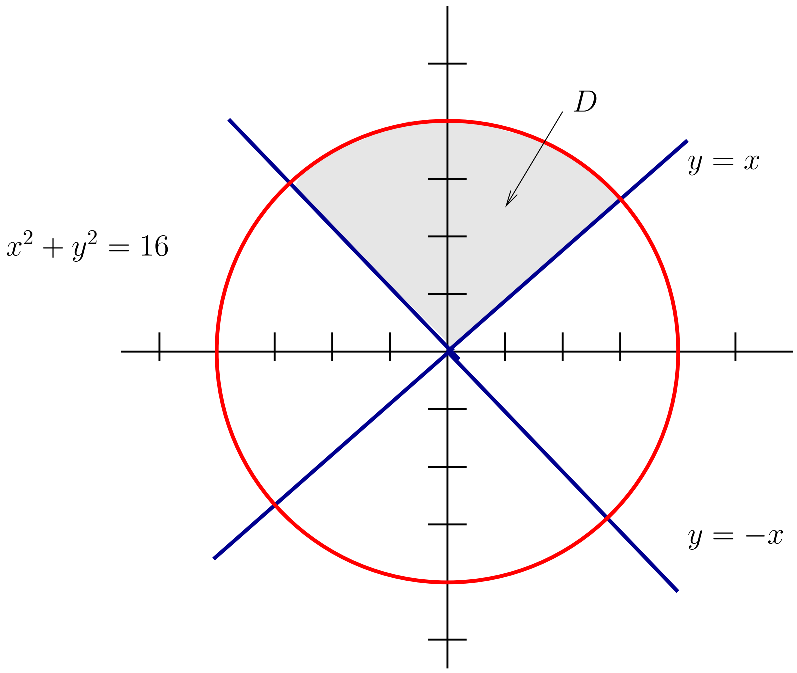

Figure 1. The region D.

(7) Let E be the tetrahedron bounded by the planes $z = 0$, $x = 0$, $y = 0$, $x+2y+z = 1$. Suppose that its density can be described by the function $\rho(x, y, z) = e^{x-y+z}$. Compute its mass.

(8) Let R be the object defined by the following bounds:

$$\begin{array}{ccccc} 1 & \leq & x & \leq & 2 \\ \ln(x) & \leq & y & \leq & e^x \\ \sqrt{x+y} & \leq & z & \leq & \sqrt{x+y+2} \end{array}$$

Use a triple integral with the change of variables:

$$\begin{array}{ccc} x & = & u \\ y & = & \ln v \\ z & = & \sqrt{w + \ln v + u} \end{array}$$

to write the volume of R as a single definite integral of a function of u.

(9) Use cylindrical coordinates to compute the volume of the region defined by:

$$
\begin{array}{rcccl}
\arcsin(y/\sqrt{x^2+y^2}) & \leq & z & \leq & \arccos(x/\sqrt{x^2+y^2}) + x^2 + y^2 \\
-x & \leq & y & \leq & x \\
-1 & \leq & x & \leq & 1
\end{array}
$$

(10) You will also be asked one of the following questions:

(a) Prove that for a unit vector $\mathbf{u}$ and a differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$:

$$f_{\mathbf{u}}(x, y) = \nabla f(x, y) \cdot \mathbf{u}$$

(b) Prove that ∇f is the direction in which f increases most quickly.

(c) Suppose that $g : \mathbb{R} \to \mathbb{R}^3$ and $f : \mathbb{R}^3 \to \mathbb{R}$ are differentiable functions. Explain why the chain rule $(f(g(x)))' = f'(g(x)) \cdot g'(x)$ works.

(d) Give an example of a function $f : \mathbb{R}^2 \to \mathbb{R}$ which has partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial g}{\partial y}$ at $(0, 0)$ but which is not differentiable at $(0, 0)$.