

Expected counts and the χ^2 test

Observed Values

	19	20	21	22	Total
Parents'	324	378	337	318	1357
Another's	37	47	40	38	162
Own Place	116	279	372	487	1254
Group Q	58	60	49	25	192
Other	5	2	3	9	19
Total	540	766	801	877	2984

H₀: The proportion of each age group living in a given location is the same as the proportion of any other age group living in the same location.

H_a: The distribution of living arrangements for different age groups is different.
i.e. There are two age groups with different proportions having the same living arrangement.

Example: If the proportions of 19, 20, 21, and 22 year olds living with their parents is the same then we would expect the proportion to be roughly the pooled proportion:

$$\text{Total \# with Parents} / \text{Total \# in Survey} = 1357 / 2984 = .45$$

If this is the case then we would expect 45% of our sample of 19 year olds to live with their parents. That is, we would expect $(.45) * (540) = 243$ to live with parents.

The formula for the expected count in each cell is:

$$(\text{row total}) * (\text{column total}) / \text{table total}$$

Expected Values

	19	20	21	22	Total
Parents'	245.57	348.35	364.26	398.82	1357
Another's	29.32	41.59	43.49	47.6	162
Own Place	226.93	321.9	336.61	368.55	1254
Group Q.	34.75	49.29	51.54	56.43	192
Other	3.44	4.88	5.10	5.58	19
Total	540	766	801	877	2984

The formula for χ^2 is:

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

So in our case:

$$\begin{aligned} \chi^2 &= \frac{(324 - 245.57)^2}{245.57} + \frac{(378 - 348.35)^2}{348.35} \\ &+ \frac{(337 - 364.26)^2}{364.26} + \dots \\ &+ \frac{(3 - 5.1)^2}{5.1} + \frac{(9 - 5.58)^2}{5.58} \end{aligned}$$

Giving us: $\chi^2 = 193.55$ with $df = (5-1)*(4-1) = 12$

and a P value around zero.